THE DISTRIBUTION OF A HEAVILY POLYDISPERSED

AEROSOL IN A TURBULENT ATMOSPHERE AT A LONG

DISTANCE FROM AN INSTANTANEOUS

POINT SOURCE

- USSR -

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THE DISTRIBUTION OF A HEAVY POLYDISPERSED AEROSOL IN A TURBULENT ATMOSPHERE AT A LONG DISTANCE FROM AN INSTANTANEOUS POINT SOURCE

[Following is a translation of an article by I. L. Karol' and A. Ya. Pressman in Inzhenerno-Fizicheskiy Zhurnal (Engineering-Physical Journal), Vol. II, No. 9, Minsk, 1959, pages 83-91.]

This work analyzes certain qualitative laws of the distribution in the atmosphere, and of the gravitational deposition to the surface of the earth, of an aerosol that is heterogeneous with respect to its particle composition and that is at a considerable distance from its source. It discusses the case of an instantaneous point source that is located at heights h above the surface of the earth; in addition, h is much greater than the thickness of the surface layer of the atmosphere.

1. Within the limits of the semi-empirical theory of turbulent diffusion, the non-stationary process of dispersion of a monodispersive aerosol in the field of a constant wind is, as is known [1], described by the solution of the following equation with constants

$$\frac{\partial q_w}{\partial t} + u \frac{\partial q_w}{\partial x} - w \frac{\partial q_w}{\partial z} = K_x \frac{\partial^2 q_w}{\partial x^2} + K_y \frac{\partial^2 q_w}{\partial y^2} + K_z \frac{\partial^2 q_w}{\partial z^2}, \qquad (1)$$

which satisfy the initial

$$q_w \big|_{t=0} = Q \delta(x) \delta(y) \delta(z-h) \qquad (2)$$

and boundary conditions

$$q_w \to 0 \quad \text{при} \quad \sqrt{x^2+y^2+z^2} \to \infty, \qquad (3)$$

$$K_z \frac{\partial q_w}{\partial z}\bigg|_{z=0} + wq_w\bigg|_{z=0} = \beta q_w\bigg|_{z=0}, \qquad (4)$$

where $z = 0$ is the surface of the earth; q_w is the volume concentration of a monodispersive aerosol all particles of which settle in the atmosphere with constant velocity (Stokes law) w; u is the constant velocity of a wind directed along the axis OX; K_x, K_y and K_z are constant coefficients of turbulent diffusion of the aerosol in horizontal and vertical directions*; $\delta(x)$ is the delta-function of Dirac; Q is the total amount of aerosol given off by the source; β is the factor of proportionality, between the flow of aerosol through the surface $z = 0$ and the volume concentration of the aerosol, which characterize the interaction of particles of the aerosol with the surface of the earth [1].

The solution of equation (1) which satisfies the conditions of (2), (3) and (4) has the form [1]:

$$q_w = \frac{Q \exp\left[-(x-ut)^2/4K_x t - y^2/4K_y t + w(h-z)/2K_z - w^2 t/4K_z\right]}{8(\pi t)^{3/2}\sqrt{K_x K_y K_z}} \times$$

$$\times \left[e^{-\frac{(z-h)^2}{4K_z t}} + e^{-\frac{(z+h)^2}{4K_z t}} - 2e^{-\frac{(z+h)^2}{4K_z t}}\omega\sqrt{\frac{t}{K_z}}\Psi\left(\frac{h+z+\omega t}{2\sqrt{K_z t}}\right)\right], \qquad (5)$$

where

$$\omega = 2\beta - w \geqslant 0\,^{**}; \quad \Psi(\zeta) = \frac{\sqrt{\pi}}{2}e^{-\zeta^2}\text{erfc}\,\zeta =$$

$$= e^{-\zeta^2}\int_0^\infty e^{-\tau^2}d\tau.$$

As in reference [2], let us assume that the aerosol, heterogenous with respect to particle composition, may be described by incomplete gamma-distribution of particles with respect to the velocities of precipitation w [3], whose density of distribution has the form:

$$N(w) = \frac{n^{n+1}}{\Gamma(n+1)w_m}\left[\frac{w}{w_m}e^{-\frac{w}{w_m}}\right]^n, \qquad (6)$$

where parameter w_m characterizes the velocity of the aerosol fraction prevailing with respect to the amount of particles; n is the degree of homogeneity of the distribution of particles of the aerosol with respect to the velocities w; G(x) is

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*K_x, K_y and K_z may be considered constant, if the source is located much higher than the thickness of the surface layer of the atmosphere.
** The condition $\omega \geqslant 0$ is sufficient for the uniqueness of the solution of the problems (1) - (4).

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the gamma function.

In distribution (6) let us note two extreme cases:

a) the aerosol is homogenous, i.e., it consists of particles which precipitate in the atmosphere with the same velocity $w = w_m$; in this case, n → infinity and (6) transforms into

$$N(w) \underset{n \to \infty}{\to} \delta(w - w_m); \qquad (7)$$

b) virtually non-precipitating particles prevail in the aerosol, i.e., $w_m = 0$; in this case, assuming $w_m = nw_0$ and transforming in (6) towards the limit with n → 0 we obtain:

$$N(w \underset{n \to 0}{\to} \frac{1}{w_0} e^{-\frac{w}{w_s}}. \qquad (8)$$

In connection with the heterogeneity of the aerosol, it is necessary to take into account the dependence of the value of β and the coefficients of turbulent diffusion on the velocities of fall of particles w.

It is natural to assume that the condition of absorption is realized for comparatively heavy particles on the surface of the earth, i.e., generally speaking $\beta(w) \underset{w \to \infty}{\to} \infty$.

On the other hand, at a great distance from the source (with respect to the wind), when the practically weightless component of the aerosol is dispersed, the interaction of the flow of the aerosol with the surface of the earth (z = 0) will be determined by the value: $\beta(w)\big|_{w=0} = \beta_0.$

We shall examine two cases:

1) $\beta_c = 0$, i.e., the "weightless" component of the aerosol is repelled by the surface of the earth;

2) $\beta_c > 0$, i.e., on the surface of the earth partial or total ($\beta_0 = \infty$) absorption of the "weightless" component of the aerosol takes place.

The coefficients of turbulent diffusion $K_{x, y, z}$ (in the same way as in [2], we will consider to be dependent on w in such a way that

$$K_{x, y, z}(w) \underset{w \to \infty}{\to} 0 \quad \text{и} \quad K_{x, y, z}(w) \underset{w \to 0}{\to} K_{x, y, z}(0).$$

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Let us give the coefficients K_x, $_y$, $_z$ smaller values, insofar as we regard the dispersion of the virtually non-precipitating component of the aerosol with large values of time t.

2. Let us begin to obtain formulas for the volume and surface concentration of a polydispersed aerosol at a large distance from the source; furthermore, let us begin from the case $\beta_0 = 0$. Assuming here the arbitrary power dependence $\beta(w) = bw^m$, from conditions $\beta(0) = 0$ and $\omega \geqslant 0$ we will find* that

$$\beta = bw. \tag{9}$$

with $b = 1$ the boundary condition (4) has the appearance:

$$K_z \left. \frac{\partial q}{\partial z} \right|_{z=0} = 0 \tag{10}$$

and signifies the absence of a turbulent flow of the aerosol through the surface $z = 0$. This boundary condition serves in problems (1) - (4) for considering the influence of the surface layer of the atmosphere on the distribution of the settled admixture on the surface of the earth.

Assuming in (5) $\beta = w = \omega$, let us write the following expression for the volume concentration of a polydispersed aerosol:

$$q(x, y, z, t) = \int\limits_0^\infty q_w N(w)\, dw =$$

$$= \left(\frac{2n}{w_m} \sqrt{\frac{K_z}{t}} \right)^{n+1} \frac{Q \exp\left[-\dfrac{(x-ut)^2}{4K_x t} - \dfrac{y^2}{4K_y t} \right]}{8\pi^{3/2} \Gamma(n+1) t^{3/2} \sqrt{K_x K_y K_z}} \times$$

$$\times \int\limits_0^\infty \lambda^n e^{2\lambda\xi_1 - \lambda^2} \left[e^{-\xi_2^2} + e^{-\xi_0^2} - 4e^{-\xi_0^2} \lambda \Psi(\xi_0 + \lambda) \right] d\lambda, \tag{11}$$

where

$$\lambda = \frac{w}{2}\sqrt{\frac{t}{K_z}}; \quad \xi_0 = \frac{h+z}{2\sqrt{K_z t}}; \quad \xi_2 = \frac{h-z}{2\sqrt{K_z t}};$$

$$\xi_1 = \xi_2 - \frac{n}{w_m}\sqrt{\frac{K_z}{t}}. \tag{12}$$

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*Actually, let $\beta = bw^m$; for fulfillment under all conditions $\omega = 2\beta - \omega = 2bw^m - w > 0$ it is necessary that m - 1.

Let us construct the expansion of the function (11) into a series with respect to powers of small (for large values of t) parameters ξ_0, ξ_1 and ξ_2; preliminarily, let us expand, with respect to powers of ξ_0 the function ψ:

$$\Psi(\zeta_0 + \lambda) = \sum_{k=1}^{\infty} (-2)^{k-1} H_{-k}(\lambda) \xi_0^{k-1}, \qquad (13)$$

where

$$H_{-k}(\lambda) = \frac{1}{\Gamma(k)} \int_0^{\infty} \zeta^{k-1} e^{-\zeta^2 - 2\lambda\zeta} d\zeta$$

is the function of Hermite [5]; after which let us transcribe (11) in the form

$$q(x, y, z, t) = \left(\frac{2n}{w_m} \sqrt{\frac{K_z}{t}} \right)^{n+1} \frac{Q e^{-\frac{(x-ut)^2}{4K_x t} - \frac{y^2}{4K_y t}}}{\Gamma(n+1) 8 (\pi t)^{3/2} \sqrt{K_x K_y K_z}} \times$$

$$\times \left[e^{\xi_1^2} \left(e^{-\xi_2^2} + e^{-\xi_0^2} \right) I_{n,0}(\xi_1) - 4 e^{\xi_1^2 - \xi_0^2} \times \right.$$

$$\left. \times \sum_{k=1}^{\infty} (-2\xi_0)^{k-1} I_{n+1,k}(\xi_1) \right], \qquad (14)$$

where

$$I_{n,0}(\xi_1) = \int_0^{\infty} \lambda^n e^{-(\xi_1-\lambda)^2} d\lambda = \sum_{s=0}^{\infty} \frac{\xi_1^s}{s!} \int_0^{\infty} \lambda^n H_s(\lambda) e^{-\lambda^2} d\lambda,$$

$$I_{n+1,k}(\xi_1) = \int_0^{\infty} \lambda^{n+1} e^{-(\xi_1-\lambda)^2} H_{-k}(\lambda) d\lambda = \qquad (15)$$

$$= \sum_{s=0}^{\infty} \frac{\xi_1^s}{s!} \int_0^{\infty} \lambda^{n+1} H_s(\lambda) H_{-k}(\lambda) e^{-\lambda^2} d\lambda.$$

Limiting ourselves in both obtained series to only the first terms, we have:

$$I_{n,0}(\xi_1) \simeq \int_0^{\infty} \lambda^n e^{-\lambda^2} d\lambda + O(\xi_1) = \frac{1}{2} \Gamma\left(\frac{n+1}{2} \right) + O(\xi_1),$$

$$\qquad (16)$$

$$I_{n+1,0}(\xi_1) \simeq \int_0^{\infty} \lambda^{n+1} H_{-1}(\lambda) e^{-\lambda^2} d\lambda + O(\xi_1) =$$

$$= \frac{1}{4} \frac{n+1}{n+2} \Gamma \left(\frac{n+1}{2} \right) + O(\xi_1).$$

Then the first term of the expansion of the function (14) with respect to powers of $1/\sqrt{t}$ will assume the form of

$$q(x, y, z, t) \simeq \left(\frac{n}{w_m} \right)^{n+1} K_z^{n/2} Q \times$$

$$\times \frac{\exp \left[-\frac{(x-ut)^2}{4K_x t} - \frac{y^2}{4K_y t} + \frac{(h-z)^2}{4K_z t} - \frac{n(h-z)}{w_m t} + \frac{n^2 K_z}{w_m^2 t} \right]}{8\pi \Gamma \left(\frac{n}{2} + 1 \right) \sqrt{K_x K_y} \, t^{(n+4)/2}} \times$$

$$\times \left[e^{-\frac{(z-h)^2}{4K_z t}} - e^{-\frac{(h+z)^2}{4K_z t}} \frac{n}{n+2} + O(\xi_0) \right]. \tag{17}$$

Let us note that term-by-term integration of the series for ψ in (13) is justified, since the series obtained by substitution of (13) into (14) agrees uniformly with respect to λ in $\lambda \geqslant 0$.

In the case under consideration, the formula for the surface concentration of an aerosol which has settled on the earth is obtained from expression (5) by the following means:

$$\sigma(x, y) = \int_0^\infty \left[K_z \frac{\partial q_w}{\partial z} + w q_w \right]_{z=0} N(w) dw = \int_0^\infty \beta(w) q_w \bigg|_{z=0} N(w) dw =$$

$$= \left(\frac{2n}{w_m} \sqrt{K_z} \right)^{n+1} \frac{Q e^{\frac{xu}{2K_x}}}{2\pi^{3/2} \sqrt{K_x K_y}} \int_0^\infty e^{-\frac{L}{t} - \frac{u^2 t}{4K_x}} t^{-(n+5)/2} \times$$

$$\times \left[I_{n+1, 0}(\xi_1) - 2 \sum_{k=1}^\infty (-L/\sqrt{K_z t})^{k-1} I_{n+2, k}(\xi_1) \right] dt, \tag{18}$$

where

$$L = \frac{x^2}{4K_x t} + \frac{y^2}{4K_y t} + \frac{nh}{w_m t} - \frac{n^2 K_z}{w_m^2 t}, \tag{19}$$

and $I_{k, \ell}(\xi_1)$ is determined by expression (15). Since we are analyzing the dispersion of the aerosol at a great distance from the source, we are interested in the value $\sigma(x, y)$ for large values of L. Therefore, having utilized the known Laplace method [5], let us construct the asymptotic expansion of the integral (18) for $L \to$ infinity. In this case, as is

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known [5], the coefficients of the asymptotic expansion (18) are determined from the expansion of the extraexponential factor of the integrand expression in (18) with respect to the negative powers of t. Thus, considering (16), we can write:

$$\sigma(x,y) = \left(\frac{n}{w_m}\sqrt{K_z}\right)^{n+1} Q \frac{e^{xu/2K_x}}{2\pi(n+3)\Gamma\left(\frac{n+1}{2}\right)\sqrt{K_xK_y}} \times$$

$$\times \int_0^\infty e^{-\frac{L}{t} - \frac{u^2t}{4K_x}} t^{-\frac{n+5}{2}} [1 + O(t^{-1/2})]\, dt =$$

$$= \left(\frac{n}{w_m}\sqrt{K_z}\right)^{n+1} Q \frac{e^{xu/2K_x} u^{(n+3)/2}}{8\pi(n+3)\Gamma\left(\frac{n+1}{2}\right)\sqrt{K_xK_y}(4K_xL)^{\frac{n+3}{4}}} \times$$

$$\times \left\{ K_{(n+3)/2}(u\sqrt{L/K_x}) + O\left[\left(\frac{u^2}{4K_xL}\right)^{1/4} K_{(n+4)/2}(u\sqrt{L/K_x})\right]\right\}, \quad (20)$$

where $K_m(z)$ is the function of Macdonald [4]. Utilizing the asymptotic expansion $K_m(z)$ for large values of argument z, we find the final expression for the main term of the asymptotic expansion $\sigma(x, y)$ for large values of L:

$$\sigma(x,y) = \left(\frac{n}{w_m}\sqrt{K_z}\right)^{n+1} \frac{Qu^{n/2+1}\exp\left[\frac{xu}{2K_x} - u\sqrt{L/K_x}\right]}{\sqrt{\pi K_y}(n+3)\Gamma\left(\frac{n+1}{2}\right)(4K_xL)^{n/4+1}} \times$$

$$\times \left[1 + O\left(\frac{\sqrt{u}}{\sqrt[4]{4K_xL}}\right)\right]. \quad (21)$$

3. Let us examine the case of partial absorption of the "weightless" component of the aerosol by the surface of the earth, i.e., the case $\beta(0) > 0$. Here, for large values of time t, the argument of the function ψ in expression (5) increases limitlessly; therefore, for construction of the asymptotic expansion $q(x,y,z,t)$ and $\sigma(x, y)$ at large distances from the source, let us utilize the asymptotic expansion of the function $\psi(\zeta)$ for $\zeta \to$ infinity [4], and let us present the extraexponential part of the third addend of expression (5) in the form:

$$\omega\sqrt{\frac{t}{K_z}}\,\psi\left(\frac{h+z+\omega t}{2\sqrt{K_z t}}\right) =$$

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$$= \frac{\omega t}{h + z + \omega t} \left(1 + \sum_{k=1}^{N} \frac{(-1)^k (2k-1)!!}{(h + z + \omega t)^{2k}} (2K_z t)^k \right) = \sum_{l=0}^{\infty} \frac{(-1)^l a_l}{t^l}, \quad (22)$$

where

$$a_l = \sum_{k=0}^{l} \frac{(2k-1)!!(l+k)!(2K_z)^k(h+z)^k}{2k!(l-k)!\, \omega^{k+l}};$$

$$a_0 = 1; \quad a_1 = \frac{h+z}{\omega} + \frac{2K_z}{\omega^2}.$$

Computing the volume concentration of a polydispersed aerosol, let us substitute this asymptotic expression for $t \to$ infinity into the third addend of the integrand expression (11) and after term-by-term integration with respect to w we shall obtain*

$$q(x, y, z, t) = \left(\frac{2n}{w_m} \sqrt{\frac{K_z}{t}} \right)^{n+1} Q \frac{\exp\left[-(x - ut)^2/4K_x t - y^2/4K_y t \right]}{8(\pi t)^{3/2} \sqrt{K_x K_y K_z}} \times$$

$$\times \left[\left(e^{-\xi_2^2} - e^{-\xi_0^2} \right) H_{-(n+1)}(\xi_1) + 2e^{-\xi_0^2} \sum_{l=1}^{\infty} (-1)^{l-1} d_l \, t^{-l} \right], \quad (23)$$

where

$$d_l = \frac{1}{\Gamma(n+1)} \int_0^{\infty} a_l \lambda^n e^{2\lambda \xi_1 - \lambda^2} d\lambda.$$

Utilizing the Laplace method, it is not difficult to write the asymptotic expansion of the coefficients d_l with $t \to$ infinity, namely

$$d_l = \sqrt{\pi}\, 2^{-(n+1)} \Gamma^{-1} \left(\frac{n}{2} + 1 \right) \times$$

$$\times \sum_{s=}^{\infty} b_s^{(l)} \Gamma \left(\frac{n+1+s}{2} \right) \Gamma^{-1} \left(\frac{n+1}{2} \right) \left(\frac{2\sqrt{K_z}}{t} \right)^s. \quad (24)$$

*Term-by-term integration in (22) with respect to w from 0 to infinity is justified, since, (23) by virtue of min $\omega = \beta/_{w=0} >$ > 0, this expansion is uniform with respect to w in w $\geq$ 0.

where the coefficients $b_s^{(l)}$ are determined from the expansion

$$a_l \exp\left[\frac{h-z}{2K_z} - \frac{n}{w_m}\right] w = \sum_{s=0}^{\infty} b_s^{(l)} w^s, \quad b_0^{(l)} = a_l\big|_{w=0}. \qquad (25)$$

Limiting ourselves, as in the previous case, by the main terms in the obtained expansions, we will find from expression (23) the main term of the asymptotic expansion q(x, y,z,t) with t → infinity:*

$$q(x, y, z, t) = \left(\frac{n}{w_m}\right)^{n+1} \frac{Q K_z^{n/2} \exp\left[-(x-ut)^2/4K_x t - y^2/4K_y t\right]}{8\pi \sqrt{K_x K_y}\,\Gamma(n/2+1)\,t^{(n+4)/2}} \times$$

$$\times \left[\left(e^{-\frac{(h-z)^2}{4K_z t}} - e^{-\frac{(h+z)^2}{4K_z t}}\right)\left(1 + O\left(\frac{1}{\sqrt{t}}\right)\right)\right]. \qquad (26)$$

The asymptotic expression for the surface concentration $\sigma(x, y)$ at large distances from the source is found in the same way in paragraph 2, and its main term has the form of

$$\sigma(x, y) = \left(\frac{n}{w_m}\right)^{n+1} \frac{Q K_z^{n/2}\, e^{xu/2K_x}}{8\pi \sqrt{K_x K_y}\,\Gamma(n/2+1)}\left(h + \frac{K_z}{\beta_0}\right) \times$$

$$\times \int_0^{\infty} e^{-\frac{L}{t} - \frac{u^2 t}{4K_x}}\, t^{-\frac{n+6}{2}}\left[1 + O\left(\frac{1}{\sqrt{t}}\right)\right] dt.$$

Carrying out the integration, and utilizing the asymptotic representation of the Macdonald function $K_m(z)$, we have, analogously to (21),

$$\sigma(x, y) = \left(\frac{n}{w_m}\right)^{n+1} \frac{Q K_z^{n/2} \exp[xu/2K_x - u\sqrt{L/K_x}]\, u^{(n+3)/2}}{4\sqrt{\pi K_y}\,\Gamma(n/2+1)(4K_x L)^{(n+5)/4}} \times$$

$$\times \left(h + \frac{K_z}{\beta_0}\right)\left[1 + O\sqrt{u/2\sqrt{L K_x}}\right]. \qquad (27)$$

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*In this case, the equality

$$H_\nu(0) = \sqrt{\pi}\, 2^\nu\, \frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)}.$$

is utilized.

Let us note that if the condition of absorption of the aerosol by the earth's surface is satisfied:

$$q\big|_{z=0} = 0,$$

i.e., in the specific case of condition (4) with $\beta = \beta_0 = \omega = $ infinity, then in (22) and (23) $d_l = a_l = 0$, $l = 1, 2, 3, \dots$ should be assumed, and (23) is essentially simplified, and the expression for $\sigma(x, y)$ will assume the appearance of*

$$\sigma(x, y) = \frac{QHK_z^{n/2}\, e^{xu/2K_x}\, u^{n/2+2}}{4\pi\sqrt{K_z K_y}\,(4K_x L)^{n/4+1}} \left(\frac{n}{w_m}\right)^{n+1} \times$$

$$\times \left[\sum_{k=0}^{\infty} \frac{u^k \zeta^{2k} (4K_x L)^{-k/2}}{2k!\,\Gamma\left(\dfrac{n}{2}+1-k\right)} K_{n/2+2+k}\left(u\sqrt{L/K_x}\right) + \right.$$

$$\left. + \sum_{k=0}^{\infty} \frac{u^{k+1/2}\zeta^{2k+1}(4K_x L)^{-k/2-1/4}}{(2k+1)!\,\Gamma\left(\dfrac{n+1}{2}-k\right)} K_{(n+s)/2+k}\left(u\sqrt{L/K_x}\right)\right]. \qquad (28)$$

where

$$\zeta = (w_m h - 2nK_z)/w_m\sqrt{K_z}.$$

4. Let us analyze the results obtained.

From formulas (21) and (27), it follows that, for large values of the value x (i.e., at large distance from the source with respect to wind direction), the surface concentration of the fallen aerosol $\sigma(x, y)$ decreases with an increase of x according to the exponential law. Indeed, for $x \to$ infinity in (21) and (27), the factors which contain x will be asymptotically equal to:

$$4K_x L = x^2 \left[1 + \frac{4K_x}{x^2}\left(\frac{y^2}{4K_y} + \frac{nh}{w_m} - \frac{n^2 K_z}{w_m^2}\right)\right] =$$

$$= x^2[1 + O(x^{-2})],$$

$$\exp\left[xu/2K_x - u\sqrt{L/K_x}\right] =$$

$$= \exp\left[xu/2K_x\left(1 - \sqrt{1 + \frac{4K_x}{x^2}\left(\frac{y^2}{4K_x} + \frac{nh}{w_m} - \frac{n^2 K_z}{w_m^2}\right)}\right)\right] =$$

$$= \exp\left[-\frac{u}{x}\left(\frac{y^2}{4K_y} + \frac{nh}{w_m} - \frac{n^2 K_z}{w_m^2}\right) + O(x^{-3})\right]. \qquad (29)$$

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*The main term (28) may also be obtained from (27), there passing to the limit for $\beta_0 \to$ infinity.

Thus, in case of repulsion of the "weightless" component of an aerosol by the surface of the earth ($\beta_0 = 0$, formula (21)), far from the source $\sigma(x, y)$ decreases as $x^{-(n+4)/2}$ and, in case of even partial absorption ($\beta_0 > 0$, formula (27)) decreases as $x^{-(n+3)/2}$. Let us note that for $n \to$ infinity, $w = w_m \neq 0$ and $\beta_0 > 0$, i.e., in case of precipitation of a heavy homogenous aerosol on the absorbing (even if partially) surface $z = 0$, $\sigma(x, y)$ decreases with an increase of x according to the exponential law (as expression:

$$x^{-2}\exp\left[-\frac{xu}{2K_x}\left(\sqrt{1+\frac{w_m^2 K_x}{u^2 K_z}}-1\right)\right] \sim x^{-2}\exp\left(-\frac{xw_m^2}{4uK_z}\right)$$

for small w_m), and not according to the square law, as is erroneously stated in reference [6]. The surface concentration of a monodispersed aerosol decreases with an increase of x as x^{-2} only in the case of $w_m = 0$ and $\beta_0 > 0$ (partial absorption of the "weightless" aerosol).

As is seen from formulas (21), (27) and expressions (29), for x infinity the main terms of the asymptotic expansions (x, y) do not depend on K_x; i.e., the turbulent diffusion in the direction of the wind does not influence the distribution of the surface concentration of an aerosol precipitated at large distance from the source, under any boundary conditions on the surface of the earth.

The main terms of the asymptotic expansions $\sigma(x, y)$ in (21) and (27) depend essentially on the value of K_z; i.e., the role of turbulent diffusion with respect to the vertical is essential in precipitation of an aerosol at large distances from the source in the direction of the wind, except for the case $\beta_0 > 0$, $n = 0$ (distribution of the type (8)), when with $w_m = nw_0$ and $n \to 0$ the main term in the right side (27) becomes independent of K_z.

Thus, some quantitative and qualitative characteristics of dispersion of a polydispersed aerosol in a turbulent medium may differ considerably from the appropriate regularities of the distribution of a homogenous aerosol, even if it is assumed (as is done in the present work, as well as in [2]) that various fractions of the polydispersed aerosol disperse independently of each other.

SUMMARY

The paper deals with qualitative and quantitative characteristics o the process of dispersion of polydisperse aerosol a long distance downwind of the source.

The relationship between the expressions for volume concentration and surface concentration of polydisperse aerosol deposited from the atmosphere and boundary conditions on the surface of the earth is derived in pp. 1 – 3. Thus, formulae for volume and surface concentrations are shown to be essentially different depending on whether or not the „weightless" component of polydisperse aerosol is reflected by the surface of the earth (Formulae 17, 20, 21 and 26, 27, 28).

A detailed qualitative analysis of the formulae obtained is given in p. 4 [of the original document].

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